**Exploring Volume Elements with Known Cross-Sections**

**Introduction**

I wanted to investigate how areas of known cross-sections can be used to calculate volumes. My focus was on shapes like squares, equilateral triangles, and semi-circles, all constructed perpendicular to the xxx-axis. To ensure precision, I decided to use MATLAB for visualization and computation.

**Methods and Computations**

I started by visualizing how cross-sectional areas could be extended along the xxx-axis to create a volume. For simplicity, I focused on three shapes: squares, equilateral triangles, and semi-circles. Here's how I approached each step in MATLAB:

**MATLAB Code: Volume with Square Cross-Sections**

matlab

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% Define the bounds for the integral

x\_start = 0; % Starting point of integration

x\_end = 4; % Ending point of integration

% Step 1: Define the function for side length (s)

% The side length s is derived from the top function minus the bottom function.

s = @(x) sqrt(x);

% Step 2: Calculate the volume for square cross-sections

% The volume element is s^2 \* delta x. I integrated over the bounds to find total volume.

volume\_square = integral(@(x) s(x).^2, x\_start, x\_end);

% Display the result

fprintf('Volume with square cross-sections: %.2f cubic units\n', volume\_square);

*Explanation*:  
I started by defining the side length (sss) as the square root of xxx, since s=x−0s = \sqrt{x} - 0s=x​−0. The volume for each slab was calculated as s2Δxs^2 \Delta xs2Δx, and integrating over xxx provided the total volume.

**MATLAB Code: Volume with Equilateral Triangle Cross-Sections**

matlab

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% Step 1: Define the height of the equilateral triangle

% Using trigonometry, the height is (sqrt(3)/2) \* s.

height\_triangle = @(x) sqrt(3)/2 \* s(x);

% Step 2: Calculate the area of the equilateral triangle cross-section

% The area is (1/2) \* base \* height.

area\_triangle = @(x) 0.5 \* s(x) .\* height\_triangle(x);

% Step 3: Calculate the volume for equilateral triangle cross-sections

% The volume element is area \* delta x. Integrate to find total volume.

volume\_triangle = integral(@(x) area\_triangle(x), x\_start, x\_end);

% Display the result

fprintf('Volume with equilateral triangle cross-sections: %.2f cubic units\n', volume\_triangle);

*Explanation*:  
Here, I used the formula for the height of an equilateral triangle, h=(3/2)⋅sh = (\sqrt{3}/2) \cdot sh=(3​/2)⋅s, and computed the area as (1/2)⋅base⋅height(1/2) \cdot \text{base} \cdot \text{height}(1/2)⋅base⋅height. The total volume was the integral of the area over the bounds.

**MATLAB Code: Volume with Semi-Circular Cross-Sections**

matlab

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% Step 1: Define the radius of the semi-circle

% The radius is half the side length, r = s/2.

radius = @(x) s(x) / 2;

% Step 2: Calculate the area of the semi-circle cross-section

% The area is (1/2) \* pi \* r^2.

area\_semi\_circle = @(x) 0.5 \* pi \* (radius(x).^2);

% Step 3: Calculate the volume for semi-circular cross-sections

% The volume element is area \* delta x. Integrate to find total volume.

volume\_semi\_circle = integral(@(x) area\_semi\_circle(x), x\_start, x\_end);

% Display the result

fprintf('Volume with semi-circular cross-sections: %.2f cubic units\n', volume\_semi\_circle);

*Explanation*:  
For semi-circles, I calculated the radius as half the side length (r=s/2r = s/2r=s/2). The area was derived from the formula (1/2)⋅π⋅r2(1/2) \cdot \pi \cdot r^2(1/2)⋅π⋅r2, and integration gave the total volume.

**Results**

1. **Square Cross-Sections**:  
   The calculated volume was 8 cubic units8 \, \text{cubic units}8cubic units, matching the integral of xxx over [0,4][0, 4][0,4].
2. **Equilateral Triangle Cross-Sections**:  
   The volume was scaled by the constant 3/4\sqrt{3}/43​/4, yielding a smaller volume than the square cross-sections.
3. **Semi-Circular Cross-Sections**:  
   The volume was scaled by π/8\pi/8π/8, producing the smallest volume among the three shapes.

**Discussion**

By varying the cross-sectional shapes, I observed how the constant factors (3/4\sqrt{3}/43​/4 and π/8\pi/8π/8) modified the total volume. These results highlight the importance of shape in determining volume while showcasing the power of MATLAB in simplifying complex integrals.